Exercise 316

For the following problems, state the domain and range of the given functions:

$$f = x^2 + 2x - 3, \qquad g = \ln(x - 5), \qquad h = \frac{1}{x + 4}$$

$$h \circ f$$

Solution

Evaluate $h \circ f$.

$$h \circ f = h(f(x)) = h(x^2 + 2x - 3) = \frac{1}{(x^2 + 2x - 3) + 4} = \frac{1}{x^2 + 2x + 1} = \frac{1}{(x + 1)^2}$$

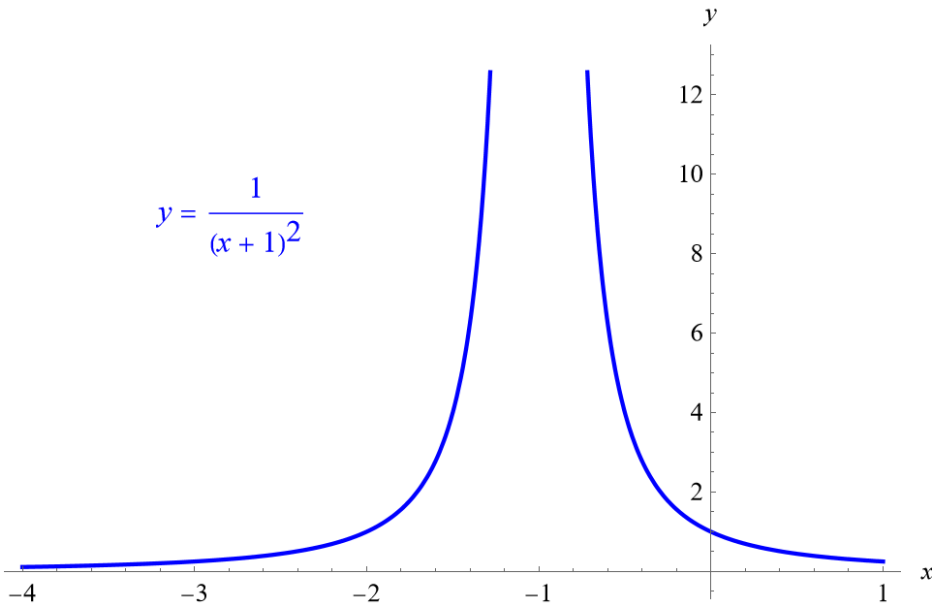
$h \circ f$ is a rational function, and the one thing to know about rational functions is that the denominator cannot be zero.

$$(x + 1)^2 \neq 0$$

$$x + 1 \neq 0$$

$$x \neq -1$$

Therefore, the domain is $\{x \mid x \neq -1\}$. Below is a graph of $h \circ f$ versus x.



The rational function takes on all positive y-values only. Therefore, the range is $\{y \mid 0 < y < \infty\}$.